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Concerning Measurements in Quantum Theory: A Critique of a Recent Proposal†

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1. *Introduction*

Measurements became a 'problem' for quantum theory essentially for the following simple reason: The quantum theoretical account of micro processes requires us on many (in fact most) occasions to represent the state of a system as a linear superposition of eigenstates.‡ The results of a measurement on such a system, however, always issue in the appearance of one particular eigenvalue associated with only one particular eigenstate among those appearing in the expansion; thus if one wants, following classical physics, the results of a measurement to refer to the state of the measured system at all (*whether predictively or retrodictively*) it seems necessary to say that at some point in time the system was (or is) in a state accurately represented by the particular single eigenstate associated with the measurement result. But this conclusion forces us to recognize the existence of a 'gap' between the two representations, between the linear superposition and the single eigenstate. The so-called 'problem of measurement' in quantum theory is to understand the origin and nature of this 'gap'.

In a recent book, Professor Josef Jauch (1968) develops an interesting treatment of this problem, and it is my purpose to submit that approach to a critical examination in this paper. But first the better-known approaches need to be briefly developed.

The most well known, one could say the 'standard', account of the gap is that due to von Neumann (1932). Von Neumann supposed that measurements had predictive value, i.e. that the state of a system for times immediately after the measurement time were to be represented by the appropriate single eigenstates. In effect, von Neumann's 'theory' of measure-

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‡ In fact no matter what the state of the system there will always be some 'observables' for which the appropriate representation of the system's state requires the introduction of such linear superpositions of the appropriate eigenstates.

ment was simply to assert that the act or process of measurement *reduced* the state of the system from whatever it was prior to the measurement time to the appropriate single eigenstate after the measurement process was completed. In the picturesque language of Hilbert space, the measurement projects the state vector onto one or other of the basis vectors. This process is the so-called *reduction of the wave packet*.

The nature of this reduction process was, however, unclear. Since von Neumann had presented an argument to the effect that any part of the world up to, but not including, the observer's actual conscious activity, could be included in the quantum description of the 'object', it seemed to many that the reducing agent in the measurement situation was the conscious activity of the human mind. (See, for example, London & Bauer, 1939; Wigner, 1962, 1963.)

For many people this subjectivist position was an unsatisfactory one for physics to espouse and alternatives were sought. Perhaps the most persuasive alternative approach is that represented by what I shall call the 'Approximationists'. This group includes earlier work by Ludwig (1967) and the contemporary work of Danieri *et al.* (1962, 1966) and Rosenfeld (1965). Their work is characterized by two features: (i) they take the reduction process seriously, i.e. they regard it as (at least in part) a substantial physical process, (ii) they attempt to offer a physical explanation, compatible with the quantum theory, of the substantial physical part of the reduction process and to argue that the remainder of the process is unproblematic. In this way they aim to solve the problem of measurement.

I shall now describe the Approximationist approach in a little more detail, for it will prove helpful in understanding Jauch's position by contrast with it. One commences by characterizing 'macro variables' and 'macro states' in a manner which is the quantum analogue of the characterization of macroscopic states in classical statistical mechanics, This characterization then represents the macroscopic measuring device as a very complex statistical aggregate of microscopic components. A measuring device characterized in this fashion is then allowed to interact with a typical micro system and the whole (i.e. the composite system) allowed to evolve in the usual unitary fashion (i.e. according to Schrödinger's equation). One then argues that the resulting state of the system, after the measurement interaction has ceased, is statistically effectively indistinguishable from a state of the system in which the various possible measurement outcomes are represented by a straightforward statistical ensemble (i.e. by a mixture). In this fashion one attempts to justify (as *pragmatically* correct) the claim that at the end of the measuring process the micro system and measuring apparatus are each in some one definite state, though which is not yet known. The transition from this situation to a particular, actual known outcome for the measurement is then held to be unproblematic since it is an epistemic transition alone, i.e. it simply represents a change in our knowledge (from ignorance of which possibility was realized to the knowledge of which one was realized).

Let us set down these two accounts of measurement schematically. We shall suppose that we commence with a micro system in the following state

$$\psi^S_{t=0} = \sum_i a_i \phi^S_{i,t=0} \tag{1.1}$$

In the traditional von Neumann account one then supposes that the measuring instrument is also in some state or other (say the null position of its pointer reading) and we shall characterize this state by $\psi^M_{t=0}$. Let the evolution of the system under the measurement interaction be governed by a unitary operator $U(t', t'')$ such that

$$U(0, t)(\psi^M_{t=0} \otimes a_i \phi^S_{i,t=0}) = a_i \phi^S_{i,t} \otimes \psi_t{}^M(i) \tag{1.2}$$

(here $\otimes$ is the tensor product). Equation (2.2) represents a good measurement interaction of the 'first' kind (i.e. the measurement does not 'disturb' the state of the system and it concludes with the apparatus state one-one correlated with the appropriate mathematical components of the object state). Initially, then, we have the state of the combined system consisting of object and measuring apparatus as

$$\psi^{S+M}_{t=0} = \psi^M_{t=0} \otimes \sum_i a_i \phi^S_{i,t=0} \tag{1.3}$$

and this state has evolved into

$$\psi^{S+M}_t = \sum_i a_i \phi^S_{i,t} \otimes \psi_t{}^M(i) \tag{1.4}$$

by the end of the measurement interaction. When the result of the measurement is known the state of the system must be represented by

$$\psi^{S+M}_{k,t} = \phi^S_{k,t} \otimes \psi_t{}^M(k) \tag{1.5}$$

for some value of k belonging to the range of values of i in the above equations. The probability that the kth term will be the appropriate one to represent the outcome of the measurement is given by $|a_k|^2$. This latter remark motivates consideration of the following mixed state in place of ψ^{S+M}_t:

$$W'^{S+M}_t = \sum_i |a_i|^2 P_{\phi^S_{i,t} \otimes \psi^M{}_t(i)} \tag{1.6}$$

Where $P_{[\phi]}$ is the projection on to the manifold [ ]. $W'^{S+M}_t \neq W^{S+M}_t$, the statistical density operator corresponding to ψ^{S+M}_t. W'^{S+M}_t does, however, yield the same particular outcomes and with the same probabilities as W^{S+M}_t for a large class of, though not all, measurements. (See on this score the work of W. H. Furry (1936a, b), in particular, and my analysis (Hooker, 1971) where other reference to the literature may be found.)

Though the von Neumann reduction ignores W'^{S+M}_t, the Approximationists essentially attempt to utilize it as a half-way house in the measurement process. The approximationist approach begins with the same state for the object system S, but an entirely different characterization of the initial state of the measuring apparatus M. (By contrast with the simple von

Neumann assumption that the measuring apparatus is in some eigenstate, usually the null state, of its pointer reading, the Approximationists attempt, at least in principle, to characterize the full atomic complexity of the measuring instrument.) Let us call this initial measuring apparatus state $\psi^M_{A,t=0}$. Under the measurement interaction the combined system also evolves unitarily, i.e. according to Schrödinger's equation, under a unitary operator which we shall call $U'(t',t'')$. At the end of the measurement interaction we say that the system has evolved into the combined state $\psi^{S+M}_{A,t}$ given by:

$$\psi^{S+M}_{A,t} = U'(0,t)\,(\psi^M_{A,t=0} \otimes \sum_i a_i \phi^S_{i,t=0}) \tag{1.7}$$

At this point the Approximationists aim to show that, after a sufficiently long time, the composite system $S+M$ can be represented by a statistical operator which is macroscopically statistically indistinguishable from one of the form of W'^{S+M}_t, call it $W'^{S+M}_{A,t}$, (the only essential difference is that the instrument states are more complex than those given in W'^{S+M}_t and are of the macro type, i.e. represented by projections on manifolds of the instrument Hilbert space rather than on a single subspace). We now also agree to accept the so-called *Ignorance Interpretation of Mixtures*, i.e. agree to claim that if a system S is in a mixture of states ψ_k with weighting coefficients ρ_k then one is entitled to assert that actually the system is in one or other of the definite states ψ_k and is in that state with probability ρ_k. If we put these latter two claims together we are licensed to claim that at the end of the measuring interaction, for all macroscopic purposes, the object and apparatus may effectively be claimed to be in some one or other of the definite states given by a mixture of the equation (1.6) type, with the probabilities $|a_k|^2$. The last part of the measuring process takes place when a human observer looks to see the actual outcome and finds which among the possibilities presented actually occurs, i.e. the transition from a representation of the equation (1.6) type to that given by equation (1.5) occurs. We may now schematically compare and contrast the two accounts of measurement in the following diagram:

von Neumann:

$$P_{\psi^{S+M}_{t=0}} \underset{I(U)}{\rightarrow} P_{\psi^{S+M}_t} \underset{R}{\rightarrow} P_{\psi^{S+M}_{k,t}}$$

Approximationist:

$$P_{\psi^{S+M}_{A,t=0}} \underset{I(U')}{\rightarrow} P_{\psi^{S+M}_{A,t}} \underset{\text{macro}}{\equiv} W'^{S+M}_{A,t} \underset{R'}{\rightarrow} P_{\psi^{S+M}_{k,t}}$$

The approximationist treatment is very much more complex than the von Neumann treatment. What the approximationist claims to gain is a more realistic account of measurement. He claims this on two counts: (i) that the subjective role of the observer is removed from the realm of *physical* processes (and restricted to the R' transition), (ii) that the measuring instrument is described quantum mechanically in a more accurate and physically plausible fashion.

The approximationist tries to limit the non-unitary changes in the representation of a system to those changes which may be accounted as *purely epistemic* and which, occurring also in classical physics, are held to be unproblematic and not to represent an intrusion of the observer's mind into natural physical processes.

Though it is not my purpose to pursue the matter here in detail, I point out that both of these approaches to measurement have been severely criticized. The former basically because of the subjective or mentalistic component it injects into physics, but also for more technical reasons. (For some of the literature see Fine, 1968, 1969; Margenau, 1950, 1958; Margenau & Park, 1967). The approximationist approach has been criticized by Jauch *et al.* (1967) (but cf. Loinger, 1968), by Feyerabend (1962) and most decisively by Bub (1968). The essence of the non-technical part of Bub's criticism lies in pointing out that the approximationist account offered by Danieri *et al.* (1962) still permits the occurrence of superposed macro states, which occurrence is simply not a conceptually coherent possibility.

What makes Jauch's account of measurement so interesting is that he aims at achieving a theory of measurement which retains the simplicity of the von Neumann approach, the physical credibility of the approximationist approach, further advantages which the approximationist approach does not have, and yet one which is free of the criticisms of both of the above approaches. It is now time to elaborate this ingenious approach.

2. *Jauch's Theory of Measurement*

In discussing Jauch we shall use the terminology developed above, which is equivalent to Jauch's, indicating correspondence between his equations and those above. To begin with we need to note the following mathematical fact. Let W_t^{S+M}, $W_t'^{S+M}$, $W_{A,t}^{S+M}$ be the statistical density operators corresponding to the states respectively of equations (1.4), (1.6) and (1.7) above. If H^S, H^M are the Hilbert spaces associated with the systems S and M respectively then the above operators (or states) refer to vectors in the composite Hilbert space $H^{S+M} = H^S \otimes H^M$. Assertion: If the composite state is characterized by the statistical density operator W_t^{S+M}, then the *reduced component statistical density operators*, W_t^S, W_t^M are given by

$$W_t^S = \sum_i |a_i|^2 P_i^S \tag{2.1}$$

$$W_t^M = \sum_i |a_i|^2 P_i^M \tag{2.2}$$

where the P_i^S, P_i^M are projection operators for one-dimensional subspaces containing $\phi_{i,t}^S, \psi_t^M(i)$ in H^S and H^M respectively.

Jauch discusses an initial state for the system to be measured of the sort given by equation (1.1) above, an initial measuring instrument state and measuring interaction operator which leads to a final composite state for object + measuring apparatus of the sort given by equation (1.4) above. Having obtained this final linear superposition state, Jauch then goes on to comment:

> After the measurement is completed we can imagine the interaction between S [the atomic system] and m [measuring instrument] removed. The reading of the scale consists in amplifying the record contained in m and deducing from it the state of S.
>
> The state of m, to be read with the amplifier, is obtained from the pure state (11–34) [↔ equation (1.4) above] by reducing that state to the system m. We use the reduction formulas of the previous section [cf. equations (2.1) and (2.2) above]. ...
>
> We see that both states have become mixtures. ... No further observation on m will modify the state, and the measurement has become an objective record. ... each individual system m which may be used in the statistic of the measurement realizes one of the ... alternatives. ... there is no question of any superposition here. The reduction of the state to the system m has wiped out any phase relations. (Jauch, 1968, pp. 183–4).

What exactly is going on here? What Jauch has to say leaves the matter somewhat obscure. There is no doubt that the state representations accorded to the reduced component states [Jauch's equation 11–35; ↔ equations (2.1) and (2.2) of text above] no longer retain any coherent phase relations among their individual terms. Equally certain is the fact that their coefficients give correct statistical weightings to the various possibilities occurring in them. In fact Jauch goes further and in the above quote affirms that we are actually entitled to say that one or other of the possibilities has actually occurred, objectively, independently of any further observation [compare his discussion of 'events' (Jauch, 1968) in section 11-6]. But then what has become of the composite system? Have we destroyed in some way the composite linear superposition when we performed the reductions to the sub-spaces? Jauch describes the process in different ways. At one point he says 'we can *imagine* the interaction ... removed' and at another point he says that '... both states have *become* mixtures'. This suggests that having the interactions cease and performing the reductions is some substantial kind of physical process, a process which destroys the original coherent linear superposition and replaces it by two incoherent statistical mixtures. But then what sort of reduction process is this? Jauch nowhere discusses the matter.

Further, he actually says things which suggest that this reduction ought *not* to be taken as representing any significant change in the composite system. A little later on in the text Jauch considers the consistency of his measuring 'theory' (in fact he considers, not the usual problem, namely what the results of the measuring process would be if the original measuring instrument were included in the quantum mechanical 'object', but the reverse question of what the results will be if the object be included in the measuring instrument). At that point he remarks 'in this case there is no occasion to reduce the pure state ... to that of a mixture'. But this suggests that the original reduction process which we considered has no physical significance, but only a purely formal one, for we apparently have a free choice about whether we employ it or no, depending on the point of view which we adopt toward the measuring interaction. But if the reduction were to be a substantial physical process, depending upon the measurement interaction ceasing, then we would not have an arbitrary choice as to whether to take it into account or not.

Many of the points raised in this paragraph will be returned to in what follows, but I believe that we shall gain a clearer insight into Jauch's intentions if we review briefly his response to the Einstein, Podolsky, Rosen (hereafter EPR) paradox.

I shall assume the basic EPR paradox familiar to the reader. [It may be found discussed in detail in Feyerabend (1962), Furry (1936a, b), Hooker (1970, 1972), and Jauch (1968) for example.] The actual form of the paradox discussed by Jauch is that due to Bohm (1951). In describing the paradox Jauch has this to say:

> Let us assume that we have two systems I and II, which at a given time can interact with each other. We assume that the states of each system are completely described by a two-dimensional vector space. Let $\phi_\pm$ represent a complete orthonormal set of vectors in the first space and $\psi_\pm$ a similar set in the second space. Let us further assume that the interaction between the two systems is such that at some time the (pure) state of the joint system is given by
>
> $$\phi = \frac{1}{\sqrt{2}}[(\phi_+ \otimes \psi_+) + (\phi_- \otimes \psi_-)] \tag{2.3}$$
>
> [$\leftrightarrow$ equation 11-38 of Jauch]
>
> We now assume that the two systems can be isolated from each other ... so that any observation carried out on one of the component systems cannot have any physical effect on the other system.
>
> After this separation the state is still given by equation (2.3). If we now measure on system I whether it is in a state ϕ_+ or ϕ_- we find that it is in $\phi_\pm$ with probability $\frac{1}{2}$. The interesting point is that a measurement of $\phi_\pm$ constitutes at the same time a measurement of $\psi_\pm$ on system II. ... since the two systems are physically separated we have a means of determining the state of system II 'without in any manner whatsoever perturbing the state' of that system. According to the criterion of Einstein, Podolsky and Rosen, the quantity with the eigenstates $\psi_\pm$ of system II must therefore have an element of physical reality. ...
>
> ... Moreover this definite value must have had the same element of reality even *before* the measurement on system I was carried out, since a measurement on system I cannot produce any physical effect whatsoever on system II and thus cannot change the reality of a physical quantity in that system.

> We are thus driven to the conclusion that the system $(\mathrm{I}+\mathrm{II})$ is in a mixture of two different states, namely, the state $\phi_+ \otimes \psi_+$ and $\phi_- \otimes \psi_-$ mixed, with probabilities $\frac{1}{2}$. Bur such a state is different from the state expressed by equation (2.3). Thus the acceptance of the notion of 'physical reality' has led us to a contradiction. (Jauch, 1968, pp. 186–7; Jauch's italics.)

Jauch replies to this problem by bringing to bear the theory of measurement which he has previously outlined (see above). In this reply he employs the quantities $P_\pm$, $Q_\pm$ which are projection operators in the Hilbert spaces of systems I and II respectively with the eigenfunctions $\phi_\pm$, $\psi_\pm$ respectively. He says:

> [The theory of measurement] ... tells us that after measurement of the quantity $P_\pm$, the system is in the state $W_\mathrm{I} \otimes W_\mathrm{II}$, where W_I and W_II are the reductions of the state $W = P_\phi$ to the sub-systems I and II respectively. This result is a direct consequence of the [preceding] analysis ...
>
> The effect of the observer on system $(\mathrm{I}+\mathrm{II})$ was thus to change the state W of that system to the state $W_\mathrm{I} \otimes W_\mathrm{II}$. This change of the state of the entire system is exactly the same as the change which would have been obtained by measuring the quantity $Q_\pm$ of system II. We now see quite clearly that the attempt at restricting the observation to system I is illusory. The effect on the entire system is exactly the same, whether we observe $P_\pm$ in system I or $Q_\pm$ in system II. To be sure, in neither case is the state of sub-system I or sub-system II modified in any manner whatsoever. This state is *before* and *after* the measurement given by W_I for I and W_II for II.
>
> The paradox originates in our habit of thinking that the states of two sub-systems determine uniquely the state of the composite system. As we have shown ... this is usually not the case. In the present example the two *different* states $W = P_\phi$ and $W_\mathrm{I} \otimes W_\mathrm{II}$ have the same reductions to systems I and II and the measurement of either $P_\pm$ *or* $Q_\pm$ changes the state W of the combined system to the state $(W_\mathrm{I} + W_\mathrm{II})$.
>
> This shows that the application of Einstein's criterion of physical reality becomes ambiguous. It all depends on how we want to interpret the conditions 'in any manner whatsoever'. If we refer it only to the state of the sub-systems I or II, it is obviously fulfilled; if we refer it to the entire system $(\mathrm{I}+\mathrm{II})$, it is not. In no case is there a contradiction of the uncertainty relation, because, as we have seen, a measurement of $P_\pm$ has exactly the same effect on the states as a measurement of $Q_\pm$. (Jauch 1968, pp. 189–90; Jauch's italics.)

There are several things about this reply to the EPR paradox which we need to note. (i) The first is that Jauch says quite clearly that the measurement does have a significant effect on the state of the composite system, for it changes that state from $W = P_\phi$ to $W_\mathrm{I} \otimes W_\mathrm{II}$. Shortly we shall examine how that change can be seen as a consequence of the previous theory of measurement which Jauch has outlined. (2) The key to Jauch's reply to EPR is the distinction which he draws between the state of the *composite* system and the states of the *component* systems. For whereas the latter do not change under the measurement process, the former does and it is precisely this difference which allows Jauch to accuse EPR of ambiguity in the application of their reality criterion. Now since even *before* the measurement was performed Jauch holds that the component states in the EPR system are given by W_{I} and W_{II} it is clear that he must subscribe to the *consistency* of holding the following: *simultaneously* the composite system can be in the state $W=P_{\phi}$ and the component systems in the states W_{I} and W_{II}. We shall later examine the consistency of this position.

But before we go on to examine various aspects of Jauch's point of view it is important to emphasize the elegance and power of the position as it now appears from his reply to EPR. For if what Jauch has to say can be upheld then he will have produced a solution to the measurement problem which completely bypasses the need either for a reduction of the wave packet or for the intricacies of the approximationist approach, and yet at the same time solves in an elegant fashion the physical heart of the EPR paradox. I shall now fill in these points in more detail.

If we are able to maintain, as Jauch evidently wishes to, that *simultaneously* with the composite system being in a linear superposition the component systems can be regarded as being represented by statistical mixtures, and hence as being in some definite state or other then we can explain immediately the occurrence of definite measuring results. For when the measuring instrument interacts with the atomic object the two become a single composite system and in general Schrödinger's equation couples these systems inextricably together in a linear superposition. It is the business of the theories of measurement previously considered to provide some de-coupling device, and this is done in a harsh fashion by von Neumann's reduction postulate and in a more sophisticated way by the Approximationist school. But Jauch is able to assert that simultaneously with the admission of the Schrödinger equation linear superposition, the component systems, i.e. the atomic object and the measuring instrument, are in reality representable by statistical mixtures and hence (ignorance interpretation) in one or other of the definite states. Looking at the measurement result merely reveals which of these definite states in fact the instrument is in (and hence which correlated definite state the atomic object is in). One seems to see immediately that there is no need for a reduction postulate of any sort and that the linear superposition given by Schrödinger's equation is held to do no more than express correlations between the components of the composite system (in this case between measuring instrument state and atomic object state) which cannot otherwise be expressed in the individual statistical mixtures for the components. In this way Jauch is able to provide a very clean and simple view of the measurement process which undercuts the need for any of the traditional complications and thereby does away with all of the problems which those complications bring with them (e.g. the subjectivism suggested by the von Neumann reduction process).

Moreover, Jauch's theory provides a very acceptable physical picture of what is happening in EPR-type situations. The physical heart of the EPR objection is the non-local properties of quantum theory which it seems to highlight. These arise essentially because the principle of superposition is valid everywhere (superselection rules excluded). Thus even when the two EPR components are spatially removed to very large distances apart the quantum mechanical representation of their composite state is still the linear superposition which seems to lead inevitably to the idea that the two are still in some substantial way connected with one another (see, for example, the argument presented in Bohm, 1951, Chapter 23 and in Hooker, 1970). It is the physical implausibility of such non-localness implied by the superposition principle that EPR was attacking. (It is precisely this feature of quantum theory which has turned out to be such an integral part of it, see, for example, the powerful conclusions drawn on this basis by Bell, 1966, cf. also Wigner, 1970.) But Jauch is able to assert that even *before* the measurement had taken place the component systems were *already* representable by statistical mixtures and hence *already* in definite states. For, taking our clue from Jauch's connecting the legitimacy of performing the reduction of the composite state to the component Hilbert spaces with the cessation of the appropriate interaction (see commencement of first quotation from Jauch, 1968, pp. 8–9 above) we conclude that immediately the EPR components cease interacting and separate it is legitimate to perform the reduction and thus arrive at the conclusion that the component systems are then already in definite states, before any measurement is performed. This provides us then with the essentially classical picture of what is happening in the EPR situation, namely that after the interaction between two components has ceased the states of the components are to be thought of as perfectly definite and *merely* correlated (i.e. correlated, but not because of any continuing physical connection between them). And measurement on one of the components then merely reveals the state of that component and hence, via the correlation, the state of the other, but no non-local connection need be postulated to understand and explain this physically. This point of view then represents a particularly elegant solution, both of the *formal* EPR objection and the *physical* problem of non-locality which EPR raises.

It is the elegance of Jauch's position which makes it worthwhile pursuing and it is to be regretted that our subsequent examination will show the position to be untenable.

3. *Critique of Jauch's Theory (I): Technical Remarks*

Jauch tells us that we obtain the state of the composite EPR system after a measurement has been performed by directly applying the theory of measurement which has been outlined in the previous section. But this application involves an essential element of ambiguity itself which needs to be brought out. Let us commence by considering the two components of some EPR-type system to be initially in the state.

$$\psi_0 = \phi_0{}^{\mathrm{I}} \otimes \psi_0^{\mathrm{II}} \tag{3.1}$$

where the components are I, II respectively and we are assuming for the sake of argument that they were initially unconnected with one another. They are then allowed to interact in a manner describable through the Schrödinger equation so as to form a composite system and the interaction is then imagined to cease and the two systems to be separated by a very large spatial distance. If this whole process is governed by the Schrödinger equation, an assumption on which both sides in the current dispute seem agreed, then we may represent the state of the system at the end of this process by

$$\psi^{\mathrm{EPR}} = \sum_i a_i \phi_i{}^{\mathrm{I}} \otimes \psi_i^{\mathrm{II}} \tag{3.2}$$

where $\psi^{\mathrm{EPR}} = V(0,t)(\phi_0{}^{\mathrm{I}} \otimes \psi_0^{\mathrm{II}})$ and $V(0,t)$ is the propagator for the interaction according to the Schrödinger equation. We assume the state of the system given by equation (3.2) to obtain up to some time t at which time a measurement on the system I of a physical quantity represented by the linear operator P is made. Formally, this measurement may be represented as a measurement on the composite system by replacing the linear operator P with the operator $P \otimes I$ where I is in this formula the unit operator in the Hilbert space of the component II. We assume the measurement interaction to be governed by a propagator $U'(t,t')$ which transforms the measuring instrument state into one which is correlated with the system states but does not alter the system state, i.e. it is a good measurement of the first kind. If we assume that initially the measuring instrument is in some null state of its pointer reading given by $\psi_t{}^M$ then we may represent the effect of the measuring interaction on the system by

$$U'(t,t')(\phi_i{}^{\mathrm{I}} \otimes \psi_i^{\mathrm{II}} \otimes \psi_t{}^M) = \phi_i{}^{\mathrm{I}} \otimes \psi_i^{\mathrm{II}} \otimes \psi_{t'}^M(i) \tag{3.3}$$

and thus at the end of the measuring interaction the full composite state is given by

$$\psi^{\mathrm{EPR}(t')} = \sum a_i \phi_i{}^{\mathrm{I}} \otimes \psi_i^{\mathrm{II}} \otimes \psi_{t'}^M(i) \tag{3.4}$$

The full composite state given by equation (3.4) is represented by a vector in the composite Hilbert space $H^{\mathrm{EPR}} = H^{\mathrm{I}} \otimes H^{\mathrm{II}} \otimes H^M$. Within this Hilbert space we are permitted at *least four different reductions* to component Hilbert spaces. Thus we may reduce the composite system to the component Hilbert spaces $H^{\mathrm{I}} \otimes H^{\mathrm{II}}$ and H^M, or to $H^{\mathrm{I}} \otimes H^M$ and H^{II}, or to $H^{\mathrm{II}} \otimes H^M$ and H^{I} or finally to H^{I} and H^{II} and H^M. There is no doubt that we are *formally* permitted to do this. The question is: What is the physical significance of this ambiguity of the reduction? The two most interesting possibilities amongst these four are the first and the last, so let us consider these for a moment. In the first case the reduced states are given by the following:

$$W_{\mathrm{I+II}} = \sum_i |a_i|^2 P_{\phi_i{}^{\mathrm{I}} \oplus \psi_i{}^{\mathrm{II}}} \tag{3.5}$$

$$W_M = \sum_i |a_i|^2 P_{\psi^M(i)} \tag{3.6}$$

where $P_{\phi_i{}^{\mathrm{I}} \oplus \psi_i{}^{\mathrm{II}}}$ is a projection operator for the state $\phi_i{}^{\mathrm{I}} \otimes \psi_i^{\mathrm{II}}$ in the Hilbert space $H^{\mathrm{I}} \otimes H^{\mathrm{II}}$ and $P_{\psi^M(i)}$ is a projection operator for the state $\psi^M(i)$ in the Hilbert space H^M. The reduced states in the case of the fourth of the above alternatives are given by

$$W_{\mathrm{I}} = \sum_i |a_i|^2 P_{\phi_i{}^{\mathrm{I}}} \tag{3.7}$$

$$W_{\mathrm{II}} = \sum_i |a_i|^2 P_{\psi_i{}^{\mathrm{II}}} \tag{3.8}$$

and by equation (3.6) above (the symbolism should by now be obvious).

The first thing we need to note is that

$$W_{\mathrm{I+II}} \neq W_{\mathrm{I}} \otimes W_{\mathrm{II}} \tag{3.9}$$

The difference between the right and left-hand sides of equation (3.9) is simply that the one but not the other, preserves correlations between the component systems I and II. Indeed, $W_{\mathrm{I+II}}$ and $W_{\mathrm{I}} \otimes W_{\mathrm{II}}$ are not even macroscopically equivalent in Jauch's sense, i.e. they do not belong to the same equivalence class. (This is again essentially because the one does, and the other does not, preserve correlations between the systems.) In the first place this result, and similar results that would be obtained if we considered the other two of the four possible reductions, should give us pause when considering the *physical significance* of these reductions. For remember that Jauch wishes to assert that simultaneously with the composite state being in a linear superposition, i.e. being represented by $W = P_{\psi^{\mathrm{EPR}}}$, the component states can be said to be represented by the statistical mixtures obtained from the reductions. But if the reductions are not only not unique, but not even equivalent (macroscopically or microscopically) then what physical sense are we to make of this claim of Jauch's? Jauch provides us with no clue as to the answer.

Secondly, a *direct* application of the theory of measurement which Jauch outlined (see above, Section 2) to the EPR situation suggests strongly that the break should be made for the purposes of reduction between the measuring instrument Hilbert space and the Hilbert space of the measured system. After all, this is the relevant interaction which is being made to cease. But this represents the first of our above reductions and not the fourth, whereas the state which Jauch assigns to the EPR system after the measurement has been made is that corresponding to the fourth reduction alternative and not the first. Since the difference between these two is non-trivial the question of how we should apply Jauch's theory of measurement to the EPR situation is non-trivially ambiguous.

But now let us attempt to work backwards and see whether we can apply the method which Jauch uses in the EPR case to every measurement situation. That is we want to be able to argue, just as Jauch did for EPR, that even *before* the measurement the system is in a perfectly definite state and that all the measurement does is to reveal what this state is. Only in this way shall we provide the necessary *physical* picture of what is happening in the measurement process to back up my previous suggestion that Jauch's position elegantly undermines the necessity for postulating some reduction process. We begin then by considering an atomic system S initially in the state

$$\phi_{s,0} = \sum_i a_i \phi_{s,i,0}$$

and we introduce a measuring instrument M at time $t = 0$ whose initial state is again the null state $\psi_0{}^M$. If once again the measurement process is effective and of the first kind then we may represent the state of the composite system $S + M$ at the end of the measurement interaction by

$$\psi_t^{S+M} = U(0, t)(\psi_0{}^M \otimes \sum_i a_i \phi_{s,i,0}) = \sum_i a_i \phi_{s,i,t} \otimes \psi_t{}^M(i) \qquad (3.10)$$

Now if we follow Jauch, we see that when the measuring interaction ceases and we perform the reduction of the composite system to the component Hilbert spaces we obtain the familiar statistical mixtures, that for the atomic system S being given by

$$W_S = \sum_i |a_i|^2 P_{\phi_{s,i,t}} \qquad (3.11)$$

Are we now still able to say that, even *before* the measurement process began, the system S was still in some definite state or other, i.e. could its state be represented by a statistical mixture? To find the answer to this we must take the reduction of the composite state of $S + M$ *before* the measuring interaction began to the component Hilbert spaces. Now the composite state of $S + M$ before the interaction commenced is given by

$$\psi_0{}^M \otimes \phi_{s,0} = \psi_0{}^M \otimes \sum_i a_i \phi_{s,i,0}$$

and the reduction of this system to the component Hilbert space of the system S alone is given by

$$W_S' = P_{\phi_{s,0}} = P_{\sum_i a_i \phi_{s,i,0}}$$

But W_S' represents a *pure state* for S and not a mixture; $W_S' \neq W_S$. In fact, Jauch's theory of measurement is forced to postulate that the measurement process issues in the transition $W_S' \to W_S$ which is exactly the equivalent of the physical part of the reduction of the wave packet postulated by von Neumann or approximated by the Approximationist school. Thus Jauch has not in general succeeded in removing the postulation of a reduction process, but only in disguising it beneath the mathematics.

Indeed, one can work his trick if one is dealing initially with a *composite* system, for then the initial system before the measurement interaction is introduced *already* represents a state in a *composite* Hilbert space and so one can already obtain reductions for its components. But as soon as one looks at the equivalent measuring theory for a single system one sees that Jauch is forced to introduce the reduction process.

Moreover, Jauch has nothing to say to us about the physical significance of this reduction process. The one clear statement which his remarks do suggest, namely that the reductions should occur when the appropriate interaction ceases, although of no help in understanding precisely what is happening physically, does lead to trouble elsewhere. For it suggests that *already* at the cessation of the interaction between the components of an EPR-type system and *before* any measurement is introduced, we should perform the reductions on that system; but then it would *already* be illegitimate to represent the state of the EPR system at the time of measurement by equation (3.2) above and it should rather be represented by $W_{\mathrm{I}} \otimes W_{\mathrm{II}}$. But it is a well-known fact that these two representations would lead to quite different statistical predictions [indeed, even the representation given by equation (3.5) differs statistically from that given by equation (3.2)—see Furry (1936a, b), and my commentary (1972)]. But this is already to raise the question of the consistency of Jauch's position to which we must now turn in detail.

4. *Critique of Jauch's Theory (II): The Consistency of the Theory*

We shall begin by considering an abstract argument. To present the argument we shall first introduce some convenient labels. I shall describe as the *Standard Theory of Quantum States* the view that physical states are represented by vectors in the appropriate Hilbert spaces and characterize individual physical systems on particular occasions. This is the standard view of most quantum theorists and it asserts that Schrödinger's equation is valid for individual physical systems as well as for ensembles of such systems. Secondly, I shall characterize as the *Reduction Assumption* the assumption that *simultaneously* with a composite system being representable by a linear superposition, the component states are representable by the statistical mixtures obtained by reducing the composite state to the component Hilbert spaces. Thirdly and finally I shall define the *Ignorance Interpretation of Mixtures* to be the view that whenever a mixture truly characterizes the state of some physical system then that physical system can be said to be actually in one or other of the definite component states occurring in the mixture (though which state will, of course, not be known except to within a certain probability). We may now present our argument as follows:

P_1: The Standard Theory of Quantum States is assumed.

Consider now a composite state represented by a linear superposition in the quantum theory, for example the state given by equation (3.2) above.

We know from the quantum theory that the reductions of that state to the component Hilbert spaces yield the statistical mixtures W_{I} and W_{II} given by equations (3.7) and (3.8) above.

P_2: The Reduction Assumption is granted.

P_3: Therefore, simultaneously with a system being represented by the state ψ^{EPR} of equation (3.2) above, the component states are represented by W_{I} and W_{II} of equations (3.7) and (3.8) above.

P_4: The Ignorance Interpretation of Mixtures is assumed.

P_5: Therefore the actual state of the component I is one or other of the ϕ_i^{I} and the actual state of component II is one or other of the ψ_i^{II}. Let these two states be respectively ϕ_k^{I}, ψ_l^{II}.

P_6: But then the state of the composite system must be given by $\phi_k^{\mathrm{I}} \otimes \psi_l^{\mathrm{II}}$ which contradicts the original assumption that it was given by ψ^{EPR} of equation (3.2) above.

P_7: Moreover, even the conclusion that the components are in definite, though unknown, states (and even if we assume these states one-one correlated), would require $W_{\mathrm{I+II}}$ (given by equation (3.5) above) as the appropriate representation for the state of the system, which still contradicts the initial assumption.

It seems clear that Jauch assumes the standard theory of quantum states. And he has stated quite explicitly (see above quotations) that he assumes the ignorance interpretation of mixtures. But I have argued above (Section 2) that Jauch's treatment of the EPR case demands that he accept the reduction assumption (for otherwise he will not be able to say that, both *before* and after the measurement, the component states were given by the appropriate statistical mixtures). It would seem, therefore, that Jauch is caught clearly in this contradiction and that it will apply to his treatment of EPR, at least for all times prior to the actual cessation of the measurement interaction. I can see no way out of this dilemma for Jauch, nor does he make any comment on how he would deal with the difficulty.

Since the argument seems clearly valid, one must remove the contradiction by denying one or more of the premises. I have considered the alternatives in some detail in my essay (Hooker, 1972) and here I shall offer only brief reactions to the alternatives.

There are those who attempt to deny the ignorance interpretation of mixtures (cf. Hooker, 1972; van Fraassen, 1971, for references). If this is not accompanied by a denial of the standard theory of quantum states then one is forced to view the attribution of a mixed state to a physical system as the attribution of some new kind of individual state which a system may possess. But this involves the renunciation of an essentially clear physical interpretation of mixed states (namely that provided by the concept of a classical ensemble) for a totally mysterious conception of the state. What are the physical features of such states? What is it to be in a mixture now? (These questions are not made any more clear by the only relevant piece of information we would have about these new mixed states, namely that they are, or can be, the component states in a composite system which is represented by a linear superposition of their appropriate component states!) If the denial of the ignorance interpretation of mixtures is accompanied by a denial of the standard theory of quantum states, then one is totally in the dark as to what is being offered until an entirely new theory is put forward. To my knowledge, no such sweeping alternative that is physically satisfactory has been proposed.

On the other hand there are those who attempt to deny the standard theory of quantum states. Thus it is becoming popular to claim that all quantum states, including linear superpositions, refer essentially only to ensemble states and that individual systems cannot be attributed a quantum state. (See, for example, Park, 1968; Ballentine, 1970). This alternative involves regarding linear superpositions as, nevertheless, representing states of statistical ensembles. But what could the individual member states of the statistical ensemble possibly be? They cannot each be represented by the entire linear superposition on pain of trivializing this particular alternative. Since, on the other hand, the outcomes of particular measurements refer to the components of the linear superposition the only plausible alternative is to say that the ensemble members have states given by the individual components of the linear superposition. Thus in the case of the superposition given by equation (3.2) above, since on a measurement of $P_{\phi^{\mathrm{I}}} \otimes Q_{\psi^{\mathrm{II}}}$ we obtain the eigenvalues corresponding to the composite eigenstate $\phi_i^{\mathrm{I}} \otimes \psi_i^{\mathrm{II}}$ with probability $|a_i|^2$ it seems necessary to say, if the original state is to represent an ensemble, that each member of the ensemble is in one or other of the states $\phi_i^{\mathrm{I}} \otimes \psi_i^{\mathrm{II}}$. But in this case we should completely obscure the difference between the state represented by equation (3.2) and that represented by equation (3.5) above. Now one might respond by saying that for measurements of other quantities it is the state given by equation (3.2) and not (3.5), which would give the correct statistics. This only leads, however, to a further difficulty in comprehension, for (3.2) gives the correct statistics for the measurements of other quantities only because of the coherence properties between the various members of the expansion in (3.2); but what on earth is the conception of an ensemble whose individual member states interfere with one another? (Moreover, the coefficients in equation (3.2) are complex and do not in general sum to unity so that they cannot be regarded as statistical weights in the usual sense.)

If we reject the former two alternatives, we are forced to deny the reduction assumption. Now I have already tried to provide physical argument for denying the reduction assumption in the preceding text. For in complex systems the reductions which are mathematically possible are neither mathematically nor physically (macroscopically or microscopically) equivalent. If the reduction assumption were taken seriously then it should at least be applicable to all possible reductions, but this is not possible since it would then lead to a contradiction. If we reject the reduction assumption, what are we to make of the formal mathematical ability to perform such reductions? There seems here a very plausible point of view to take. It is the full composite system which is the most faithful representation of the *physical* state of affairs. What the reductions do is to provide *partial information* concerning *some aspects* of the states of the component systems in such a composite system. It must be emphasized that it is *information only* and that it is information which pertains only to the *responses of such component systems to a measurement interaction.* For if we take the quantum theory seriously we shall be forced to conclude from the symmetry properties of the composite system and the many differing reductions which are in general possible for it, differing according to the kind of measurement proposed, that the components in a composite system really have no separately assignable physical state; one can only obtain information about the outcomes of measurements upon them. This view seems both consistent and more physically plausible than its alternatives and it is the one I propose we should adopt.

Having now discussed the consistency of Jauch's view—and having found it to be inconsistent—we turn briefly to discuss the physical plausibility of Jauch's position.

5. *Critique of Jauch's Theory (III): The Physical Reasonableness of the Theory*

The main point here, which has been examined in detail elsewhere (Hooker, 1970), can be put quite simply: We have seen that Jauch must assert that the interaction between one of the components of the EPR system and a measuring instrument effects a significant change in the state of the *entire* system, namely it effects the change from the state $W = P_\phi$ to the state $W_{\mathrm{I}} \otimes W_{\mathrm{II}}$ (see Section 2 above). But *ex hypothesi*, the two component systems of the EPR system are no longer in physical interaction with one another. How then could a measurement on *one of the components* affect the state of the *entire composite system.* The reduction assumption would of course permit us to say that the state of the other component (for that matter, also the state of the measured component) had not really altered. But we have seen that the reduction assumption must be rejected. Since Jauch fails to provide us with any significant physical connection between the two components which would explain the transition which he is forced to postulate we are drawn back to the conclusion that there is a non-local feature to the quantum theory (in this case Jauch's quantum theory) for which we have as yet no satisfactory physical understanding.† But it was precisely this feature of quantum theory to which EPR wished to draw attention.

† Jauch sometimes speaks as if the correlations themselves provided such a connection, but these may be *mere* correlations without a physical connection as basis—see Hooker (1970). And if the only physical feature which the linear superposition of the composite state contributes are these correlations, it again obscures the difference between it and the corresponding statistical mixture, e.g. between ψ^{EPR} of Equation (3.2) and $W_{\mathrm{I+II}}$ of equation (3.5).

Before concluding this section let us now return briefly to a question left somewhat vague in the preceding: Under just which conditions is it permissible to perform the reductions of a composite system state to the component Hilbert spaces? The answer initially suggested by a first consideration of Jauch's treatment of EPR, and by some of his earlier comments, is that whenever the appropriate interaction ceases we are to perform the reductions and to take the state of the composite systems thereafter as the tensor product of the component states thus obtained. But we have seen that this process would lead us to conclude, for example, that after the interaction had ceased among the EPR components, but before a measurement was being made, the true state of the system would be that given by $W_{\mathrm{I}} \otimes W_{\mathrm{II}}$ [or perhaps data given by equation (3.5) above]; but neither this system [nor that of equation (3.5)] can predict the correct statistics for measurements of other physical quantities on the composite system, this can only be done by the representation given in equation (3.2) [again see Furry (1936a, b) and my commentary (1972)]. Nor are we able to hold that the components are in the reduced states simultaneously with the composite state holding, for this is just the reduction assumption. Perhaps we ought to say, on Jauch's account, that these reductions can be made only after a measurement interaction has taken place, and not just any interaction whatever. But why a measurement interaction? Jauch's theory gives no reason to believe that these are special in this respect. Indeed, the whole drift of the theory is that these reactions are precisely *not* special in this respect, but simply some examples of quantum interactions. We are left, therefore, with an essential element of obscurity in the application of Jauch's reductions, an obscurity which reflects the obscurity of what exactly one is doing *physically* when one carries out these reductions mathematically. Neither have we been able to clarify the question of when the composite state is to be considered as truly composite and when not, *vis-à-vis* the inclusion of one part of it in the other (for example *vis-à-vis* the question considered briefly on pp. 94–95 above of including the object system in the measuring apparatus). Since it would seem that when we choose to do this and when not is a purely pragmatic decision on our part, the question of when the reductions ought to be performed and when not becomes a pragmatic decision on our part. In Jauch's theory, we are left, therefore, with two elements of obscurity in the question of precisely which conditions suffice for carrying out these reductions and what their physical significance is in each case. (Of course, if one accepts the interpretation of reductions which I advocated on pp. 104–105 above then it becomes irrelevant when one performs these 'reductions', since one is not thereby doing anything *physical*, one is only obtaining information.)

6. *Conclusion*

Jauch's theory of measurement, if satisfactory, would have provided an elegant and very simple solution to all of the standard difficulties found in other versions of the measurement process in quantum theory. We have examined this theory and found it to contain an important element of obscurity (that concerned with just exactly under what conditions the reductions may be performed) and ambiguity (namely concerning which reductions are to be performed), but most importantly we have concluded that it was both inconsistent and lacking in physical plausibility.

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